Complex inflaton field in quantum cosmology

A. Yu. Kamenshchik[1†], I. M. Khalatnikov[1,2†] and A. V. Toporensky[3*]

[1] *L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygin str. 2, Moscow, 117334, Russia*

[2] *Tel Aviv University, Tel Aviv University, Raymond and Sackler Faculty of Exact Sciences, School of Physics and Astronomy, Ramat Aviv, 69978, Israel*

[3] *Sternberg Astronomical Institute, Moscow University, Moscow, 119899, Russia*

We investigate the cosmological model with the complex scalar self-interacting inflaton field non-minimally coupled to gravity. The different geometries of the Euclidean classically forbidden regions are represented. The instanton solutions of the corresponding Euclidean equations of motion are found by numerical calculations supplemented by the qualitative analysis of Lorentzian and Euclidean trajectories. The applications of these solutions to the no-boundary and tunneling proposals for the wave function of the Universe are studied. Possible interpretation of obtained results and their connection with inflationary cosmology is discussed. The restrictions on the possible values of the new quasi-fundamental constant of the theory–non-zero classical charge– are obtained. The equations of motion for the generalized cosmological model with complex scalar field are written down and investigated. The conditions of the existence of instanton solutions corresponding to permanent values of an absolute value of scalar field are obtained.

PACS: 98.80.Hw, 04.60.Kz, 98.80.Bp

† Electronic mail: kamen@landau.ac.ru

† Electronic mail: khalat@itp.ac.ru

* Electronic mail: lesha@sai.msu.su

I. Introduction

It is widely recognized that inflationary cosmological models give a good basis for the description of the observed structure of the Universe [1]. Most of these models include the so called inflaton scalar field possessing non-zero classical average value which provides the existence of an effective cosmological constant on early stage of the cosmological evolution. On one hand inflationary cosmology has received the strong support due to discovery of the anisotropy of the microwave background radiation [2] while on the other hand it is connected with such an exiting field of modern theoretical physics as quantum cosmology. The main task of quantum cosmology is the consideration of the Universe as a unique quantum object which can be described by the wave function of the Universe, obeying to the Wheeler-DeWitt equation. Studying this wave function of the Universe one can hope to get the probability distribution of the initial conditions for the Universe.

During the last decade quantum cosmology has been developing intensively on the basis of two proposals for the boundary conditions for the wave function of the Universe: the so-called "no-boundary" [3] and "tunneling" [4] proposals. Both these proposals use the apparat of Euclidean quantum field theory combined with the ideas of the theory of quantum tunneling transitions and instantons. However, these proposals taken in tree semiclassical approximation cannot provide the

normalizability of the wave function of the Universe [5] and to predict the initial conditions for the cosmological evolution providing sufficient amount of inflation [6].

One can look for different ways out from this situation. Consideration of the wave function of the Universe in one-loop approximation [7] gives us an opportunity to obtain the normalizability of the wave function of the Universe and the existence of the suitable probability distribution for the initial conditions for inflation provided proper particle content of the theory is chosen.

Another possible direction of the development of quantum cosmology is the consideration of more wide theories than traditional scheme with a real scalar field. Thus in the series of recent papers [8,9] the model with a complex scalar inflaton field was studied. One of the reasons for the consideration of a complex scalar field consists in the fact that such fields and the non-Abelian multiplets of scalar fields appear naturally in the modern theories of particle physics.

The most natural representation of the complex scalar field has the form

$$\phi = x \exp(i\theta), \tag{1.1}$$

where x is the absolute value of the complex scalar field while θ is its phase. This phase is cyclical variable corresponding to the conserved quantity – a classical charge of the Universe, which plays the role of the new quasi-fundamental constant of the theory [8].

Appearance of this new constant essentially modifies the structure of the Wheeler-DeWitt equation. Namely, the form of the superpotential $U(x, a)$, where a is the cosmological radius of the Universe displays now a new and interesting feature: the Euclidean region, i.e. the classically forbidden region where $U > 0$, is bounded by a closed curve in the minisuperspace (x, a) for a large range of parameters. Thus, in

contrast with the picture of "tunneling from nothing"[4] and with the "no-boundary proposal" for the wave function of the Universe [3] we have Lorentzian region at the very small values of cosmological radius a and hence a wave can go into Euclidean region from one side and outgo from the other. These new features of the model require reconsideration of traditional schemes [3,4] and give some additional possibilities.

Here, it is necessary to stress that speaking about the "Euclidean" or "classically forbidden" regions one should understand that in the case of the quantum gravity and cosmology these terms can be used only in "loose" sense, because due to indefiniteness of the supermetric the "Euclidean" region is not impenetrable for Lorentzian trajectories. It is well-known that in the cosmological models with inflaton scalar field the Lorentzian trajectories can penetrate into Euclidean region just like as the trajectories corresponding to the Euclidean equations of motion can leave Euclidean region for the Lorentzian one (see, for example, Refs. [10−−12]. However, one can use such terms as Euclidean and Lorentzian regions as people usually do investigating the tunneling type processes in cosmology [3−−12] and in instanton physics [13] even in the case of indefinite supermetric. Moreover, one can give to the term "Euclidean region" quite definite value as the region where the points of the minimal contraction and maximal expansion of the Universe can exist (cf. Sec. V of the present paper).

In principle, investigating the cosmological models with the complex scalar field one can use instead of parametrization (1.1) the pair of real scalar fields, representing real and imaginary part of ϕ as it was done in [14]. However, in this case the presence of the symmetry corresponding to new constant is hidden and results obtained in [14] for the case of flat Freedman-Robertson-Walker model coincide with those obtained

for the case of the real scalar field [11].

In recent years quite a few papers were devoted to the investigation of the cosmological models with the non-minimal coupling between inflaton scalar field and gravity [15,7]. On one hand such models give a lot of opportunities for matching with the observational data and ideas of particle physics, on the other hand they can be treated as more consistent from the point of view of quantum gravity [16].

In our recent letter [17], we have considered non-minimally coupled complex scalar field. It was shown that inclusion of non-minimal coupling makes the model more rich. In particular the geometry of Euclidean regions have a large diversity and depends on the choice of parameters of the theory. In the model with the minimally coupled complex scalar field it was shown that one instanton solution exist [8]. This instanton solution can be continued into the Lorentzian region in accordance with Lorentzian equations of motion and thus to provide the beginning of the inflation. This scheme gives the strong preference to the "no-boundary" proposal for the wave function of the Universe[9]. At the same time in the case of non-minimally coupled complex scalar field we can have a couple of instantons one of which is suitable for the "no-boundary" wave function of the Universe while the other is suitable for the tunneling one. This paper will be devoted to the more detailed investigation of the properties of the model represented in [17].

Here it is necessary to tell that strictly speaking the Hartle-Hawking or "no-boundary"proposal can not be literally applied to the case of complex scalar field[9]. Indeed, in the semiclassical approximation and in a minisuperspace model "no-boundary"proposal can be boiled down to the path integration from $a = 0$ up to some small value of the cosmological radius a over compact metrics and regular matter fields. However, in the presence of the phase variable θ and connected with

it new constant– charge of the Universe – the centrifugal term arises in the superpotential which prevents regularity of matter fields at $a = 0$. Thus we need in some extention of the Hartle-Hawking proposal. Throughout this paper we shall use the extention of no-boundary proposal discussed in [9]. The idea consists in the consideration that Hartle-Hawking proposalis equivalent to the requirement that the wave function of the Universe should be exponentially growing in the classically forbidden region and should be in semiclassical approximation proportional to

$$\exp(-I),$$

where I is the corresponding classical Euclidean action. Instantons considered in Refs. 8,9,17 corresponds to just this definition of the "no-boundary" wave function of the Universe.

It is necessary to add that another approach to the complex scalar field in cosmology was discussed in [18−−21] mainly in the context of wormhole solutions. It was noticed [19], that "nonzero charge can play the same role in wormhole dynamics that nonzero angular momentum does in the dynamics of a particle in an attractive central potential, or nonzero magnetic charge in the dynamics of an 't Hooft - Polyakov monopole. Nonzero angular momentum keeps the particle from falling into the origin; nonzero magnetic charge keeps the monopole from decaying into mesons; nonzero charge flowing down the throat keeps the wormhole from punching off two disconnected manifolds". Some differences between our approach and that of Refs. [18−−21] will be discussed below.

The structure of our paper is as follows: in Sec. II we obtain the new fundamental constant of our model – classical charge –and discuss its influence on the structure of the superpotential and on the tunneling process; in Sec. III we consider

the different forms of Euclidean region depending on the choice of parameters; in Sec. IV we present the equations of motion for our theory and describe the results of numerical search of instantonic solutions of Euclidean equations of motion and their connection with different versions of the boundary conditions for the wave function of the Universe; in Sec. V we present the equations of motion for the generalized model of the complex scalar field interacting with gravity and study their properties; Sec. VI is devoted to the brief summary of the obtained results.

II. Complex scalar field and the new quasi-fundamental constant

We shall consider the model with the following action:

$$S = \int d^4x\sqrt{-g}\left(\frac{m_P^2}{16\pi}(R-2\Lambda)+\frac{1}{2}g^{\mu\nu}\phi_\mu^*\phi_\nu \right.$$
$$\left.+\frac{1}{2}\xi R\phi\phi^* - \frac{1}{2}m^2\phi\phi^* - \frac{1}{4!}\lambda(\phi\phi^*)^2\right). \tag{2.1}$$

Here ξ is the parameter of non-minimal coupling (we choose for convenience the sign which is opposite to the generally accepted), λ is the parameter of the self-interaction of the scalar field, Λ is cosmological constant, m is the mass of the scalar field. The complex scalar field ϕ can be represented in the form given by Eq. (1.1).

We shall consider the minisuperspace model with the spatially homogeneous variables a (cosmological radius in the Freedman- Robertson-Walker metric), x and θ. In terms of these variables the action (2.1) looks as follows:

$$S = 2\pi^2\int dtNa^3\left(\frac{m_P^2}{16\pi}\left[6\left(\frac{\dot{a}^2}{N^2a^2}+\frac{\ddot{a}}{N^2a}+\frac{1}{a^2}\right)-2\Lambda\right]+\frac{1}{2N^2}\dot{x}^2\right.$$
$$\left.+\frac{1}{2N^2}x^2\dot{\theta}^2+3\xi\left(\frac{\dot{a}^2}{N^2a^2}+\frac{\ddot{a}}{N^2a}+\frac{1}{a^2}\right)x^2-\frac{1}{2}m^2x^2-\frac{1}{4!}\lambda x^4\right), \tag{2.2}$$

where N is the lapse function. Now, by integrating by parts one may get rid of the

terms containing $\ddot{a}$ and write down the action (2.2) in more convenient form

$$S = 2\pi^2 \int dtN \left(\frac{m_P^2}{16\pi} \left[6 \left(-\frac{\dot{a}^2 a}{N^2} + a \right) - 2\Lambda a^3 \right] + \frac{1}{2N^2}\dot{x}^2 a^3 + \frac{1}{2N^2} x^2 \dot{\theta}^2 a^3 \right.$$
$$\left. +3\xi \left(\frac{-\dot{a}^2 a}{N^2} + a \right) x^2 - 6\xi \frac{\dot{a}\dot{x}a^2 x}{N^2} - \frac{1}{2} m^2 x^2 a^3 - \frac{1}{4!}\lambda x^4 a^3 \right). \tag{2.3}$$

Let us notice that the phase variable θ is the cyclical one and correspondingly, its conjugate momentum p_θ should be conserved. We shall call its value by a charge of the Universe and shall denote it by Q

$$p_\theta = Q = a^3 x^2 \dot{\theta}. \tag{2.4}$$

Now, coming to the canonical formalism and using the relation (2.4) one can rewrite the action (2.3) in the following form:

$$S = 2\pi^2 \int dt(p_a \dot{a} + p_x \dot{x} - N\mathcal{H}), \tag{2.5}$$

where super-Hamiltonian $\mathcal{H}$ has the following form

$$\mathcal{H} = -\frac{p_a^2}{24a\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)} - \frac{\xi p_x p_a x}{2a^2\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)}$$
$$+\frac{p_x^2}{2a^3} \frac{\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2}\right)}{\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)} - U(a,x). \tag{2.6}$$

The function $U(a,x)$ which we shall call the superpotential looks as follows:

$$U(a,x) = a\left(\frac{m_P^2}{16\pi}(6 - 2\Lambda a^2)) + 3\xi x^2 \right.$$
$$\left. -\frac{Q^2}{a^4 x^2} - \frac{1}{2} m^2 x^2 a^2 - \frac{1}{24}\lambda x^4 a^2 \right). \tag{2.7}$$

The variation of the action in respect to lapse function N gives us the super-Hamiltonian constraint

$$\mathcal{H} = 0, \tag{2.8}$$

whose quantum analog is the well-known Wheeler-DeWitt equation.

Now it is convenient to go back to Lagrangian formalism and to write down the Lagrangian, depending only on two minisuperspace variables a and x and their derivatives (the lapse function N is chosen to be equal to 1):

$$\begin{aligned}L = \Bigg(& \frac{m_P^2}{16\pi}\left[6\left(-\dot{a}^2 a + a\right) - 2\Lambda a^3\right] \\ & + 3\xi\left(-\dot{a}^2 a + a\right)x^2 - 6\xi\dot{a}\dot{x}a^2 x + \frac{1}{2}\dot{x}^2 a^3 \\ & - \frac{Q^2}{2a^3x^2} - \frac{1}{2}m^2x^2a^3 - \frac{1}{4!}\lambda x^4 a^3\Bigg).\end{aligned} \tag{2.9}$$

This Lagrangian gives us the equations of motion for the minisuperspace variables a and x which will be written down and investigated in Sec. IV. (Let us notice that the simple substitution of the value of $\dot{\theta}$ from Eq. (2.4) with the subsequent variation of (2.2) in respect to a and x does not gives us the correct equations of motion).

It is clear that the transition from the Lorentzian equations of motion which can be obtained from the Lagrangian (2.9) to their Euclidean counterparts should be done by simple change of sign before terms containing time derivatives of a and x. However, if we make the transition to Euclidean (imaginary) time *before* we have got rid of $\dot{\theta}$ due to Eq. (2.4) we shall have *another* couple of Euclidean equations of motion. These equations of motion shall differ from the previous one by the sign before terms containing the new charge Q. The question arises: what pair of equations is "correct" and what is the reason of the "discrepancy"? These problems were discussed in [18] and illustrated by the example of the particle in the central potential, where the angular momentum plays the role of charge while cyclical angular variable plays the role of phase. Assuming that the correct Euclidean equations of motion are those which are obtained when the transition to Euclidean

time is carried out *after* the exclusion of cyclic variables author of Ref. 18 proposed two ways to cure the situation.

One proposal consists in the idea that it is necessary to add to the Euclidean action where all the variables are on equal footing the term which representing Lagrange multiplier multiplied by constraint

$$\frac{dQ}{d\tau}, \tag{2.10}$$

where τ is Euclidean time. Indeed, the addition of the term proportional to (2.10) to the action changes the Euclidean equations of motion in a desirable way. However, to our mind the very procedure of adding of (2.10) is an unnecessary and illegitimate simultaneously. It is unnecessary, because the Lagrangian of the theory implies the conservation of charge (i.e. the momentum conjugate to cyclic variable). Moreover, it is illegitimate because, the term (2.10) contains the second derivative of our variable θ and its addition to the action is equivalent to the forceful change of sign before terms containing Q. (One can say also that if the procedure of addition of (2.10) is legitimate then analogous procedure can be carried out with Lorentzian equations of motion as well and we shall have apparently incorrect signs before some terms in these equations).

Another idea uses the known rules for matching the solutions of classical Lorentzian equations of motion with their Euclidean counterparts [12,22]. To satisfy the principle of minimal action it is necessary not only to require the dynamical variables obey to Lorentzian and Euclidean equations of motion along the Lorentzian and Euclidean sections of their trajectories correspondingly, but also to choose the point of matching of Lorentzian and Euclidean trajectories in such a way to provide the vanishing of the first derivatives of these variables in this point. If it is impossible, then the

complexification of the trajectories is inescapable and in the points of matching the following conditions should be satisfied:

$$Re\dot{q}_E = Im\dot{q}_L; \;\; Im\dot{q}_E = -Re\dot{q}_L. \tag{2.11}$$

Now looking at the Eq. (2.4) it is easy to understand that $\dot{\theta}$ cannot vanish and thus if we want to include θ into the set of our minisuperspace variables and to treat it on equal footing with x and a then the complexification at least of the phase variable is necessary. In this case at coming to Euclidean region and at the transition from Lorentzian equations of motion to Euclidean ones it is necessary using the conditions (2.11) to go from the real phase θ to the complex one having imaginary component. Carrying this procedure out simultaneously with the transition to Euclidean (imaginary) time we again obtain the Euclidean equations of motion which coincide with those obtained from the effective Lagrangian (2.9) including only two variables a and x.

Summing up one can say that Euclidean equations of motion which are obtained from the Lorentzian one *before* exclusion of phase variable θ coincide with those obtained *after* exclusion θ provided the complexification of phase was carried out. Such procedure is quite correct from the mathematical point of view, however the appearance of an imaginary phase can evoke some difficulties connected with the interpretation.

¿From our point of view there is not necessity to consider the imaginary phases at the transition to the Euclidean equations of motion. It is much more reasonable to think that the phase variable θ does not subject to the transition from the Lorentzian time to the Euclidean time and during the tunneling transition continues to "live" in Lorentzian time. In the framework of such an approach we naturally

obtain the effective Lagrangian (2.9) and the corresponding Euclidean equations of motion. This approach accepting the simultaneous evolution of different variables in Lorentzian and Euclidean times can find justification in the fact that the very idea of introduction of Euclidean equations of motion and instantons is connected with the impossibility to describe the process of tunneling in terms of classical trajectories. However, not all the variables of the quantum system which undergoes the process of tunneling should be treated on equal footing. Some of them are not involved into the process of tunneling and their evolution can be described in terms of Lorentzian equation of motion. This fact is well-known in the non-relativistic quantum mechanics [23]. Moreover, the existence of the variables which do not undergo the tunneling transition while the system in whole does, can be used for the definition and measurement of time spent by system under barrier [24], because these "non-tunneling" variables can be used as a "quantum clock" [25].

The problem consists in the fact that there is not a procedure of subdividing of the degrees of freedom of the system under consideration onto ones which undergo tunneling and those which do not. This problem is rather complicated even in non-relativistic multidimensional quantum mechanics let alone the quantum gravity, where one additionally encounters the problem of indefiniteness of supermetric. However, in the situation when we have cyclical variables the problem becomes more simple because, we can express these variables through other ones and conserved quantities reducing in such a way the problem of tunneling to that of lower dimensionality.

Thus, in the rest of our paper we shall think that the phase θ does not undergo through tunneling and can be safely expressed through new quasi-fundamental constant - classical charge Q and variables x and a through relation (2.4). Dynamics

of these variables is described by the Lagrangian (2.9) and will be studied in Sec. IV. In the next section we shall consider the geometry of the so-called Euclidean or "classically-forbidden" regions which can be determined as the regions of the positivity of the superpotential $U(a,x)$.

III. Geometry of Euclidean regions

We have already mentioned that the very notion of the Euclidean region for the multidimensional problems and especially in the problems with the indefinite supermetric as in the case of quantum gravity and cosmology becomes "fuzzy". Nevertheless, we shall use this terminology, because it is generally accepted. Moreover, we shall show in the Sec. V that the notion of Euclidean region has the quite well-defined physical sense.

Thus we shall call Euclidean region that one where the following condition takes place:

$$U(a,x) > 0. \tag{3.1}$$

Correspondingly the boundary of this region is given by the equation

$$U(a,x) = 0. \tag{3.2}$$

Resolving this equation we can get the form of the Euclidean region in the plane of minisuperspace variables (a,x). It is interesting to compare the form of these regions for different values of parameters included on the superpotential U (Eq. (2.7)).

In the simplest case then $Q = \Lambda = \lambda = \xi = 0$ we have non-compact Euclidean region bounded by hyperbolic curve $x = \pm\sqrt{\frac{3}{4\pi}}\frac{m_P}{ma}$ (see Fig. 1a). Inclusion of the

cosmological term $\Lambda \neq 0$ implies the closing of the Euclidean region "on the right" at $a = \sqrt{\frac{3}{\Lambda}}$ (see Fig. 1b).

Inclusion of the non-zero classical charge of the scalar field $Q \neq 0$ implies the closing of the Euclidean region "on the left" and we have obtained "banana-like" structure of this region [8] (see Fig. 1c).

After the inclusion of the small term describing the non-minimal coupling between scalar field and gravity ($\xi \neq 0$) we obtain the second Euclidean region in the upper left corner of the plane (x, a) [16]. This new region is non-compact and unrestricted from above (see Fig. 1d). While increasing the value of the parameter ξ this second Euclidean region drops down and at some value of ξ . joins with the first banana-like Euclidean region (see Fig.1e). It is easy to find this value of ξ in the absence of self-interaction of the scalar field:

$$\xi = \frac{16\pi^2 m^4 Q^2}{27 m_P^4}$$

When with the growing of the value of ξ we shall have the unified Euclidean region. The boundary of this unified region is partially convex, partially concave (see Fig. 1f) and after further increasing of ξ it becomes convex (see Fig. 1g).

After inclusion of self-interaction of the scalar field $\lambda \neq 0$ we can have, depending on the values of the parameters Q, λ, ξ and m, various geometrical configurations of the Euclidean regions. It is easy to estimate the condition of closing of the Euclidean region from above: It is

$$\xi < \left(\frac{Q\lambda}{48}\right)^{2/3}$$

If this condition is satisfied only one closed "banana-like" Euclidean region exists (see Fig. 1h). In th opposite case we have two options depending on the complicated

interrelation between parameters ξ, m and λ. First, one can have two non-connected Euclidean regions (the corresponding picture is close to that described on Fig. 1d) : banana-like one and “bag-like” Euclidean region with an infinitely long narrow throat (the curves bounding the upper Euclidean region are asymptotically clinging to the ordinate axis). Second, one can have one open above bag-like Euclidean region which again has an infinitely long narrow throat (see Fig. 1i). Thus, we have seen that the inclusion of the charge Q, non-minimal coupling $\xi \neq 0$ and self-interaction of the scalar field implies a large variety of possible geometries of Euclidean regions in minisuperspace.

IV. Equations of motion, instantons and initial conditions for inflation

Now using the Lagrangian (2.9) and choosing the gauge $N = 1$ we can get the following equations of motion:

$$\begin{aligned} &\frac{m_P^2}{16\pi}\left(\ddot{a} + \frac{\dot{a}^2}{2a} + \frac{1}{2a} - \frac{\Lambda a}{2}\right) + \frac{\xi \dot{a}^2 x^2}{4a} + \frac{\xi \ddot{a} x^2}{2} + \xi x \dot{x} \dot{a} + \frac{\xi \dot{x}^2 a}{2} \\ &+\frac{\xi x \ddot{x} a}{2} + \frac{\xi x^2}{4a} + \frac{a \dot{x}^2}{8} - \frac{m^2 x^2 a}{8} + \frac{Q^2}{4a^5 x^2} - \frac{\lambda x^4 a}{96} = 0 \end{aligned} \tag{4.1}$$

and

$$\begin{aligned} &\ddot{x} + \frac{3\dot{x}\dot{a}}{a} - \frac{6\xi x \ddot{a}}{a} - \frac{6\xi \dot{a}^2 x}{a^2} \\ &-\frac{6\xi x}{a^2} + m^2 x - \frac{2Q^2}{a^6 x^3} + \frac{\lambda x^3}{6} = 0. \end{aligned} \tag{4.2}$$

Besides we can write down the first integral of motion of our dynamical system which can be obtained from the super-Hamiltonian constraint (2.8):

$$\begin{aligned} &-\frac{3}{8\pi} m_P^2 a \dot{a}^2 - 3\xi a \dot{a}^2 x^2 - 6\xi x \dot{x} \dot{a} a^2 \\ &+\frac{a^3}{2}\dot{x}^2 - U(a, x) = 0. \end{aligned} \tag{4.3}$$

It is obvious that Euclidean counterparts of the Eqs. (4.1)–(4.3) can be obtained by the changing of sign before terms containing time derivatives.

Numerically integrating the Euclidean analog of the system of equations (4.1)–(4.2) we can investigate the question about the presence of instantons. Under instantons we shall understand solutions of Euclidean equations of motion which have vanishing velocities $\dot{x}$ and $\dot{a}$ on the boundaries of the Euclidean regions which are given by Eq. (3.2). In our recent letter [16] we have investigated numerically this question and have found instanton solutions at some configurations of the Euclidean regions. Here we would like to complement the numerical investigation the equations of motion of our system by some qualitative analysis.

First of all it makes sense to resolve the equations (4.1)–(4.2) in respect with the second derivatives $\ddot{a}$ and $\ddot{x}$. The obtained expressions look as follows:

$$\begin{aligned}\ddot{a} = {} & \frac{1}{\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)} \times \left(-\frac{\dot{a}^2}{a}\left(\frac{m_P^2}{32\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)\right. \\ & -\frac{\dot{x}^2 a(4\xi+1)}{8} + \frac{\xi \dot{a}\dot{x}x}{2} - \frac{m_P^2}{32\pi a} + \frac{m_P^2 \Lambda a}{32\pi} \\ & -\frac{\xi x^2(12\xi+1)}{4a} + \frac{m^2 x^2 a(4\xi+1)}{8} - \frac{Q^2(4\xi+1)}{a^5 x^2} \\ & \left.+\frac{\lambda x^4 a(8\xi+1)}{96}\right); \end{aligned} \tag{4.4}$$

$$\begin{aligned}\ddot{x} = {} & \frac{1}{\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)} \times \left(-\frac{3m_P^2 \dot{x}\dot{a}}{16\pi a}\right. \\ & -\frac{3\xi(1+4\xi)\dot{x}\dot{a}x^2}{2a} + \frac{3m_P^2 \xi \dot{a}^2 x}{16\pi a^2} - \frac{3\xi(1+4\xi)\dot{x}^2 x}{4} \\ & +\frac{3m_P^2 \xi x}{16\pi a^2} + \frac{3\xi^2 x^3}{2a^2} - \frac{m_P^2 m^2 x}{16\pi} \\ & +\frac{\xi m^2 x^3}{4} + \frac{m_P^2 Q^2}{8\pi a^6 x^3} - \frac{\xi Q^2}{2a^6 x} \\ & \left.-\frac{m_P^2 \lambda x^3}{96\pi} - \frac{\lambda \xi x^5}{48} + \frac{3m_P^2 \xi \Lambda x}{16\pi}\right). \end{aligned} \tag{4.5}$$

Looking at Eq. (4.3) one can easily see that in the Euclidean region, where

$U(a,x) > 0$ it is possible to have $\dot{a} = 0$ i.e. that the cosmological radius can achieve extremum a_{min} or a_{max}. It is interesting to learn which points of Euclidean region can play role of points of minimal contraction and which ones can be points of maximal expansion. To understand that it is necessary to put in Eq. (4.4) $\dot{a} = 0$ and to express $\dot{x}$ as a function of x and a from Eq. (4.3). We shall have

$$\begin{aligned}\ddot{a} = &\frac{1}{\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)} \times \left(-\frac{m_P^2(1+3\xi)}{8\pi a}\right. \\ &+\frac{m_P^2 \Lambda a(1+2\xi)}{16\pi} - \frac{\xi x^2(1+6\xi)}{a} + \frac{m^2 x^2 a(1+4\xi)}{4} \\ &\left.-\frac{\lambda x^4 a(1+6\xi)}{48}\right). \end{aligned} \tag{4.6}$$

Apparently, if we have at some point $\ddot{a} > 0$ this point would correspond to the possible minimal contraction of the Universe while $\ddot{a} < 0$ corresponds to the possible maximal expansion. We can obtain the curve separating the points of possible minimal contraction from those of possible maximal expansion by putting the right-hand side of Eq. (4.6) equal to zero. In the simplest case when all the parameters besides m are equal to zero we reproduce the simple hyperbolic curve

$$x = \pm\sqrt{\frac{1}{2\pi}}\frac{m_P}{ma},$$

separating the points of minimal contraction from those of maximal expansion which was discussed in paper [10] and in also in paper [11], where it was written down in terms of phase space. This curve repeat that of the hyperbole

$$x = \pm\sqrt{\frac{3}{4\pi}}\frac{m_P}{ma},$$

separating the Euclidean region from the Lorentzian one and differs from it by the multiplicative factor $\sqrt{\frac{2}{3}}$.

It is interesting to mention that the form of the curve given by Eq. (4.6) does not depend of the charge Q.

One can get also an analogous curve separating the points of possible maximum and minimum values of the absolute value of scalar field x. These points can exist only in Lorentzian region $U(x,a) < 0$ as one sees from Eq. (4.3). Putting in eq. (4.5) $\dot{x} = 0$ and expressing $\dot{a}$ through variables x and a by resolving Eq. (4.3) we have got the following equation:

$$\begin{aligned}\ddot{x} = &\frac{1}{\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2} + 3\xi^2 x^2\right)\left(\frac{m_P^2}{16\pi} + \frac{\xi x^2}{2}\right)} \\ &\left(\frac{m_P^4 \xi \lambda x}{64\pi^2} + \frac{3m_P^2 \xi^2 x^3}{32\pi a^2} + \frac{3\xi^3 x^5}{4a^2}\right. \\ &-\frac{m_P^4 m^2 x}{256\pi^2} + \frac{\xi^2 m^2 x^5}{8} + \frac{m_P^2 Q^2}{128\pi^2 a^6 x^3} \\ &+\frac{m_P^2 Q^2 \xi}{16\pi a^6 x} - \frac{\xi^2 Q^2 x}{4a^6} - \frac{m_P^4 \lambda x^3}{1536\pi^2} \\ &\left.-\frac{m_P^2 \xi \lambda x^5}{192\pi} - \frac{\lambda \xi^2 x^7}{96} + \frac{3m_P^2 \xi^2 \Lambda x^3}{32\pi}\right). \end{aligned} \tag{4.7}$$

In the simplest case when only $m \neq 0$ the separating curve has an extremely simple form

$$x = 0.$$

Thus all the points $x > 0$ can play the role of points of possible maximum value of x.

Here it is important to add that Eqs. (4.6)–(4.7) can be used also for the analysis of Euclidean equations of motion as well. In this case the points of maximal expansion and minimal contraction can be placed only in Lorentzian region while the points where x can take minimal and maximal values can exist only in Euclidean region.

Now, recalling that an instanton is the solution of the classical Euclidean equations of motion which begins and ends at the boundary of Euclidean and Lorentzian regions with vanishing velocities $\dot{a}$ and $\dot{x}$ one can notice that instantonic trajectory should cross both separating curves (in a and in x) at least once. Thus, studying the location of separating curves we can get some information about possible existence and form of instantons.

One can easily see that in the case of the simplest model with real scalar field with or without cosmological constant Λ (see Fig. 1a) instantons cannot exist (except the trivial case $x = 0$. However, in the case when the non-zero charge Q is switched on, both separating curves (we shall call them x - separating curve and a - separating curve intersect the Euclidean field and instanton can exist. And it really exists as the numerical calculation confirms (see Fig. 2a). Using the end point of instanton on the right part of the boundary of Euclidean "banana - like" region as an initial point for the Lorentzian trajectory one can see that this Lorentzian trajectory has quasi-inflationary behaviour [8,9,17].

Let us now consider the model with non-minimally coupled complex scalar field. To begin with we choose the value of ξ is small enough and we have two disconnected Euclidean regions (such situation was presented on Fig. 1d). On the Fig. 2b depicted not only the configuration of Euclidean regions but also the form of $a-$ and x- separating curves. One can see that as in the previous case instanton can exist in the closed "banana-like" region and it does exist and can provide the suitable initial conditions for the beginning of inflationary lorentzian trajectory. At the same time another solution of Euclidean equations of motion with vanishing initial and final velocities is present. This solution connects different Euclidean regions and at first glance looks rather strange because it goes through

Lorentzian region while the more habitual instantons used to intersect Euclidean regions. However, the Lorentzian trajectory beginning from the final point of this instanton has a rather peculiar non-inflationary behaviour and hardly can be used for the description of quantum tunneling of the Universe from nothing.

Then while non-minimal coupling constant ξ is growing the path covered by this "peculiar " instanton is getting smaller and at the moment when two region meet in one point (see Fig. 1e) this second instanton degenerates into the point and disappear.

With further increasing of ξ we have one Euclidean region open above. In this case if ξ is not very high we have two instantons which lie inside of the Euclidean region [17] and intersect both separating curves (see Fig. 2c). Lower instanton corresponds to the local maximum of absolute value of action of the Euclidean trajectories going through Euclidean region, while upper instanton correspond to the local minimum of the absolute value of action. It is necessary to add that taking the growing initial values of x on the boundary of the Euclidean region under consideration (above the second instanton) we shall get the Euclidean trajectories with the unboundedly growing action. Both the end points of instantons can be used as initial points of Lorentzian trajectories having quasi-inflationary behaviour (see Fig. 2c).

It is important to notice that the upper instanton corresponding to the minimum of the absolute value of Euclidean action provides the existence of the peak of the probability distribution in the tunneling wave function of the Universe [4], because this function in the tree-level approximation has the behaviour

$$\Psi_T \sim \exp(-|I|).$$

At the same time the lower instanton corresponding to maximum of the absolute value of Euclidean action provides the existence of the peak of the probability distribution in the Hartle- Hawking wave function of the Universe [3], having behaviour

$$\Psi_T \sim \exp(+|I|).$$

Thus if we choose no-boundary proposal for the wave function of the Universe we should use the end point of lower instanton for the definition of the most probable initial condition for the cosmological evolution while if we choose tunneling proposal for the wave function of the Universe we should use the end point of upper instanton to fix the most probable initial boundary conditions for inflation.

As was already shown in [17] in the case when the parameter ξ is large and the boundary of the Euclidean region has a convex form (see Fig. 1g) we do not have instantons at all. It is quite understandable now because for such a choice of parameters the x-separating curve does not go through Euclidean region (see Fig. 2d) and the necessary conditions of existence of instantons are not satisfied. It is interesting to notice that in both these cases (Fig. 2c and Fig. 2d) due to the indefinite increasing of the absolute value of action with the increasing of the initial value of scalar field x at right boundary between Lorentzian and Euclidean regions the tunneling wave function of the Universe is normalizable already in tree-level approximation (cf. Refs. [5,7]).

Now we can investigate the instantonic solutions and initial conditions for the inflation for the case when self-interaction of inflaton field ($\lambda \neq 0$) is taken into account. As was described in the preceding section three possible configurations of Euclidean regions can exist. In the case when only one closed Euclidean region exists (see Fig. 1h) one can find only one instanton as usual. In the case when we have

two disjoint Euclidean regions we can discover two instantonic configurations: one going through closed “banana-like” and one “peculiar” connecting the boundaries of two Euclidean regions and going through Lorentzian region. The configuration of these instantons closely resembles that of Fig. 2b.

The most interesting is the situation when we have one open above Euclidean region (see Fig. 1i). In this case depending on values of parameters ξ, λ, Q and m one can observe three different situations. In the first case depicted on Fig. 2e we have two instantons corresponding to the minimum and maximum of absolute value of Euclidean action. In the second case we do not have instantons at all. For this configuration the large value of the parameter of non-minimal coupling ξ is essential. The absolute value of action is growing monotonically with the increasing of the initial value of scalar field x on the right boundary of Euclidean region, and tends to some fixed value (this value is finite in contrast with the case of the scalar field without self-interaction).

And in the third case we have only one instantonic solution corresponding to the maximum of absolute value of Euclidean action. To realize this case it is necessary to have the value of the constant of self-interaction λ large enough to provide the decreasing of the absolute value of action at the increasing of initial value of x, however, this value of λ must not to be too large to escape the closing of the Euclidean region from above.

It is interesting also to consider the case when the cosmological constant disappears ($\Lambda = 0$). The disappearance of the cosmological constant implies the opening of the Euclidean region on the right. The most interesting situation occurs when we have one Euclidean region open above and from the right (see Fig. 2f) Here instead of two instantons we have only one: that corresponding to the minimum of

the absolute value of the Euclidean action and correspondingly to the probability peak in the tunneling wave function of the Universe. The second “lower” instanton (cf. Fig. 2c) turns into the trajectory infinitely travelling through Euclidean region without reaching its boundary with Lorentzian region (see Fig. 2f). Thus in this case only the tunneling wave function of the Universe can predict the most probable initial conditions for the beginning of the cosmological evolution in contrast with the situation when we have only one closed Euclidean region where the Hartle-Hawking wave function of the Universe is preferable.

The received instantonic trajectories giving us the initial conditions for the inflationary stage of cosmological evolution can be studied from the point of view of the restrictions on the parameters of the model which can be obtained from phenomenological considerations. Here we shall discuss the restrictions on the parameters of the model provided we choose the tunneling prescription for the wave function of the Universe and we wish to have the sufficient number of e-foldings (usually it is taken equal to 60). One can show using numerical investigations that for the model without self-interaction ($\lambda = 0$) such restrictions have the following form:

$$\xi < 0.002; \ ; \ \ Q < \frac{50 m_P^2}{m^2}.$$

In the case when we have self-interaction ($\lambda \neq 0$) the restrictions have the more complicated form. First, to provide the beginning of the upper instanton more high it is necessary to have the following connection between ξ, λ and Q:

$$\xi \sim \frac{1}{48\pi} \frac{\lambda m_P^2}{m^2}.$$

And to escape the closing of the Euclidean region from above one should have

$$Q < \frac{1}{\sqrt{48\pi}} \frac{\lambda m_P^2}{m^3}.$$

V. Generalized equations of motion and separating curves

Let us consider the cosmological model with the complex scalar field and an arbitrary form of the interaction between this field and gravity, arbitrary form of the potential of scalar field and arbitrary form of the kinetic term for the scalar field as well, The action of such a model looks like that

$$S = \int d^4x\sqrt{-g}\left(RU(\phi) + \frac{1}{2}G(\phi)g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\right). \tag{5.1}$$

In the framework of the minisuperspace model considered in the present paper the action (5.1) has the following form

$$\begin{aligned} S = 2\pi^2 \int dt a^3 \left(6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} + \frac{1}{a^2}\right)U(x)\right. \\ \left. +\frac{1}{2}G(x)(\dot{x}^2 + x^2\dot{\theta}^2) - V(x)\right) \end{aligned} \tag{5.2}$$

Conservation rule for the derivative of phase θ now turns into

$$G(x)a^2x^2\dot{\theta} = Q. \tag{5.3}$$

The effective Lagrangian for the minisuperspace variables a and x is

$$\begin{aligned} L = -6a\dot{a}^2U(x) - 6a^2\dot{a}\dot{x}U'(x) + 6aU(x) \\ +\frac{G(x)a^3\dot{x}^2}{2} - \frac{Q^2}{2G(x)a^3x^2} - a^3V(x). \end{aligned} \tag{5.4}$$

¿From the Lagrangian (5.4) one can obtain the following equations of motion:

$$\begin{aligned} 12aU\ddot{a} + 6U\dot{a}^2 + 12aU'\dot{a}\dot{x} + 6a^2U''\dot{x}^2 + 6a^2U'\ddot{x} \\ +6U + \frac{3}{2}Ga^2\dot{x}^2 + \frac{3Q^2}{2Ga^4x^2} - 3a^2V = 0 \end{aligned} \tag{5.5}$$

and

$$
\begin{aligned}
&Ga^3\ddot{x} + 3Ga^2\dot{a}\dot{x} - 6aU'\dot{a}^2 - 6a^2U'\ddot{a} \\
&+\frac{G'a^3\dot{x}^2}{2} - 6aU' - \frac{Q^2}{Ga^3x^3} - \frac{Q^2G'}{2G^2a^3x^2} + a^3V' = 0.
\end{aligned}
\tag{5.6}
$$

It is more convenient to have these equation resolved in respect with accelerations $\ddot{x}$ and $\ddot{a}$:

$$
\begin{aligned}
&\ddot{x} = \frac{1}{UG+3U'^2} \times \Bigg(-\frac{3GU\dot{a}\dot{x}}{a} - \frac{6U'^2\dot{a}\dot{x}}{a} + \frac{3UU'\dot{a}^2}{a^2} \\
&-3U'U''\dot{x}^2 - \frac{3GU'\dot{x}^2}{4} - \frac{UG'\dot{x}^2}{2} + \frac{3UU'}{a^2} - \frac{3Q^2U'}{4Ga^6x^2} \\
&+\frac{3VU'}{2} + \frac{Q^2U}{Ga^6x^3} + \frac{Q^2UG'}{2G^2a^6x^2} - UV' \Bigg);
\end{aligned}
\tag{5.7}
$$

$$
\begin{aligned}
&\ddot{a} = \frac{1}{UG+3U'^2} \times \Bigg(-\frac{UG\dot{a}^2}{2a} - \frac{3U'^2\dot{a}^2}{a} + \frac{GU'\dot{a}\dot{x}}{2} - \frac{GU''a\dot{x}^2}{2} \\
&-\frac{G^2a\dot{x}^2}{8} + \frac{aG'U'\dot{x}^2}{4} - \frac{UG}{2a} - \frac{3U'^2}{a} - \frac{Q^2}{8a^5x^2} \\
&+\frac{aVG}{4} - \frac{Q^2U'}{2a^5x^3G} - \frac{Q^2U'G'}{4G^2a^5x^2} + \frac{aU'V'}{2} \Bigg).
\end{aligned}
\tag{5.8}
$$

(5.8) Now the first integral of motion of our system looks like that:

$$
\frac{Ga^3\dot{x}^2}{2} - 6aU\dot{a}^2 - 6a^2U'\dot{a}\dot{x} - \mathcal{U} = 0,
\tag{5.9}
$$

(5.9) where the superpotential $\mathcal{U}$ is

$$
\mathcal{U} = 6aU - a^3V - \frac{Q^2}{2Ga^3x^2}.
\tag{5.10}
$$

(5.10) The equation

$$
\mathcal{U} = 0
\tag{5.11}
$$

gives us the boundary between Euclidean region ($\mathcal{U} > 0$) and Lorentzian one ($\mathcal{U} < 0$).

Now let us study the turning points for a and x. It is easy to see from Eq. (5.9) that the condition $\dot{a} = 0$ can be satisfied only if $\mathcal{U} > 0$ i.e. in Euclidean region. Thus, we have physically meaningful definition of Euclidean region: it is the part of the minisuperspace where one can find the turning points (the points of minimal contraction $\dot{a} = 0, \ddot{a} > 0$ and the points of maximal expansion $\dot{a} = 0, \ddot{a} < 0$ for a cosmological radius a). Moreover, one can see that this property of Euclidean regions can be generalized for the more wide class of cosmological models than minisuperspace models consided here. The point is that the turning points are points which correspond to extrema of the conformal factor (i.e. cosmological radius) of the model under consideration. However, the conformal factor i is the only variable the squared velocity of which is included into the Lagrangian with the negative sign. Thus, the general structure of Wheeler-DeWitt equation as well as the general structure of the Lagrangian imply the fact that extrema of conformal factor are located inside the Euclidean region.

To find the curve separating the possible points of minimal contraction from those of maximal expansion we can substitute the condition $\dot{a} = 0$ into Eq. (5.9) and we find that

$$\dot{x}^2 = \frac{2\mathcal{U}}{2Ga^3}. \tag{5.12}$$

Substituting the condition $\dot{a} = 0$ and the $\dot{x}^2$ from Eq. (5.12) into Eq. (5.8) we have got

$$\begin{aligned}\ddot{a} = \frac{1}{UG + 3U'^2} \times \Bigg(& -\frac{6UU''}{a} + VU''a + \frac{Q^2U''}{2Ga^5x^2} \\ & -\frac{2GU}{a} + \frac{3UU'G'}{Ga} - \frac{VG'U'a}{2G} - \frac{Q^2G'U'}{2G^2a^5x^2} \\ & -\frac{3U'^2}{a} - \frac{Q^2U'}{2Ga^5x^3} + \frac{aU'V'}{2} + \frac{aVG}{2}\Bigg).\end{aligned} \tag{5.13}$$

In the region where right-hand side of Eq. (5.13) is positive and hence $\ddot{a} > 0$ we can

find the points of minimal contraction of the Universe while the region where right-hand side of Eq. (5.13) is negative corresponds to the points of maximal expansion. The investigation of the turning points for x can be carried out in a similar way. The main difference is that the points of maximum and minimum of x can be found only in Lorentzian region $\mathcal{U} < \prime$ as might be easily deduced from Eq. (5.9). Thus the curve separating the possible points of minimal and maximal x is described by the equation

$$\ddot{x} = \frac{1}{UG + 3U'^2} \times \left(2U'V - U'V - \frac{Q^2 U'}{2Ga^6 x^2} + \frac{Q^2 U}{Ga^3 x^3} + \frac{Q^2 G' U}{2G^2 a^6 x^2} \right). \tag{5.14}$$

Now let us consider some simple particular cases. If $G = constant, U = const$ then the equation for the curve separating the points of minimal contraction from those of maximal expansion has a very simple form:

$$a^2 = \frac{4U}{V}. \tag{5.15}$$

It worth noticing that the curve separating the Euclidean region from Lorentzian one in this case has the form

$$a^2 = \frac{6U}{V}. \tag{5.16}$$

Thus the curve (5.15) repeats the form of the curve (5.16) with the multiplicative factor 2/3. The simplest case when

$$U = m_P^2/16\pi; V = m^2 x^2/2; Q = 0$$

have already been studied in the preceding section.

The investigation of the Eq. (5.14) is of a special interest because if we find the solutions of this equation which has the form

$$x = x_0, \tag{5.17}$$

where x_0 is a constant which is independent of a then we have separating curve which is parallel to the axis a and plays role of a real tunneling geometry [26]. Indeed, we can consider the solution of Euclidean equations of motion beginning in the point $a = 0, x = x_0$ with the velocities $\dot{a} = 1, \dot{x} = 0$. Arriving at the point on the boundary $(x = x_0, a = a_0(x_0))$ with the vanishing velocities $\dot{x} = 0, \dot{a} = 0$ this solution can be called instanton and can be continued into the Lorentzian region along the Lorentzian equations of motion. To begin with let us consider the case of a real scalar field when

$$Q = 0$$

. The equation which we study has in this case a very simple form

$$2U'V - UV' = 0. \tag{5.18}$$

For the case when $U = const$ the equation (5.18) boils down to the trivial equation

$$V' = 0. \tag{5.19}$$

Now let

$$U = \frac{m_P^2}{16\pi} + \frac{\xi x^2}{2}$$

and

$$V = \Lambda + \frac{m^2 x^2}{2} + \frac{\lambda x^4}{24}.$$

Eq. (5.18) in this case turns into

$$2\xi\Lambda x - \frac{m_P^2 m^2 x}{16\pi}\frac{\xi m^2 x^3}{2} - \frac{m_P^2 \lambda x^3}{96\pi} = 0. \tag{5.20}$$

The nontrivial solution of this equation is

$$x = \pm\sqrt{\frac{\frac{m_P^2 m^2}{16\pi} - 2\xi\Lambda}{\frac{\xi m^2}{2} - \frac{m_P^2\lambda}{96\pi}}}. \tag{5.21}$$

If we choose $\lambda = 0$ and $\Lambda = 0$ when our solution will be

$$x = \pm m_P/\sqrt{8\pi\xi}. \tag{5.22}$$

It is also very interesting that we can obtain real tunneling constant in x solutions of Euclidean equations of motion which can begin from any point on the axis x. Indeed, if

$$V = \alpha U^2, \tag{5.23}$$

where α is an arbitrary constant then it is easy to see that the right-hand side of the Eq. (5.18) is identically equal to zero. The simplest realization of such a situation can be obtained by the assumption

$$V \sim x^{2p},\ U \sim x^p,$$

where p is an arbitrary real number.

Moreover, in the case when $Q \neq 0$ we can get the "horizontal" instantonic solutions adding to the requirement (5.22)) the additional one

$$G = \beta U/x^2, \tag{5.24}$$

where β is an arbitrary constant. One can check that in this case Eq. (5.18) is again satisfied identically. However, in this case we should begin instanton solution from the left branch of the boundary between Euclidean and Lorentzian regions in the minisuperspace. Apparently the choice

$$V \sim x^4,\ U \sim x^2,\ G = const$$

satisfies the conditions (5.22) and (5.23)

All these horizontal instantons can be treated as examples of real tunneling geometries and can be continued into the Lorentzian region along the Lorentzian

equations of motion. These Lorentzian trajectories represent the so-called “eternal” inflation and are of not large interest from phenomenological point of view. However, the little deviation from the corresponding initial conditions can supply as with long but not eternal inflation and can be used for the realistic scenario of the cosmological evolution.

VI. Conclusion

We have considered the cosmological model with non-minimally coupled complex scalar inflaton field. The complexity of the scalar field is boiled down on the level of the minisuperspace consideration to the inclusion into the theory a new quasi-fundamental constant – classical charge of the Universe. The presence of this constant essentially modifies the geometry of the so-called Euclidean or classically forbidden regions and correspondingly changes the physics of the tunneling transition or the “quantum birth from nothing of the Universe”.

Moreover, consideration of such a model allows us to make some predictions for the most probable initial conditions for the classical evolution of the Universe. Both the main proposal for the wave function of the Universe – no-boundary [3] and tunneling [4] are investigated. It is interesting that for different choices of parameters different proposal for the wave function of the Universe are more reasonable. Indeed, in the cases when we have only closed euclidean region one can define the Hartle-Hawking wave function of the Universe predicting some preferable value for initial evolution of the Universe; in the case when we have one non-compact Euclidean region open from above and inside this region we have two instantons both proposals for the wave function of the Universe are plausible; and at last if we

have one non-compact Euclidean region open from above and from the right (in the case when the cosmological constant Λ vanishes) only tunneling wave function of the Universe has the chances for the explanation of the beginning of the inflation.

ACKNOWLEDGMENTS

We are grateful to A.O. Barvinsky, R. Kolb, G.V. Lavrelashvili, D.N. Page, V.A. Rubakov, M.V. Sazhin, A.A. Starobinsky and A. Vilenkin for useful discussions. This work was supported by Russian Foundation for Fundamental Researches via grants No 96-02-16220 and No 96-02-17591 and by INTAS via project 93-3364-Ext.

1. A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, 1990) and references therein.

2. J. Smoot et al., *Aph. J.* **396**, L1 (1992); I. Strukov et al., *Pis'ma Astron. Zh.* **18**, 387 (1992).

3. J.B. Hartle and S.W. Hawking, *Phys. Rev.* **D28**, 2960 (1983); S.W. Hawking, *Nucl. Phys.* **B239**, 257 (1984).

4. A. Vilenkin, *Phys. Lett.* **117B**, 25 (1982); *Phys. Rev.* **D27**, 2848 (1983); *Phys. Rev.* **D30** 509 (1984); *Phys. Rev.* **D37**, 888 (1988); A.D. Linde, *Zh. Eksp. Teor. Fiz.* **87**, 369 (1984) [*Sov. Phys. JETP* **60**, 211 (1984)]; Ya.B. Zeldovich and A.A. Starobinsky, *Pis'ma Astron. Zh.* **10**, 323 (1984) [*Sov. Astron. Lett.* **10**, 135 (1984)]; V.A. Rubakov, *Phys. Lett.* **148B**, 280 (1984).

5. S.W. Hawking and D.N. Page, *Nucl. Phys.* **B264**, 185 (1986).

6. L.P. Grishchuk and L.V. Rozhansky, *Phys. Lett.* **B208**, 369 (1988); **B234**, 9 (1990).

7. A.O. Barvinsky and A.Yu. Kamenshchik, *Class. Quantum Grav.* **7**, L181 (1990); *Phys. Lett.* **B332**, 270 (1994); A.O. Barvinsky, *Phys. Rep.* **230**, 237 (1993); A.Yu. Kamenshchik, *Phys. Lett.* **B316**, 45 (1993).

8. I.M. Khalatnikov and A. Mezhlumian, *Phys. Lett.* **A169**, 308 (1992); I.M. Khalatnikov and P. Schiller, *Phys. Lett.* **B302**, 176 (1993).

9. L. Amendola, I.M. Khalatnikov, M. Litterio and F. Occhionero, *Phys. Rev.* **D49**, 1881 (1994).

10. D.N. Page, *Class. Quantum Grav.* **1**, 417 (1984).

11. V.A. Belinsky, L.P. Grishchuk, Ya.B. Zel'dovich and I.M. Khalatnikov, *J. Exp. Theor. Phys.* **89**, 346 (1985); V.A. Belinsky and I.M. Khalatnikov, *J. Exp. Theor. Phys.* **93**, 784(1987).

12. J.J. Halliwell and J.B. Hartle, *Phys. Rev.* **D41**, 1815 (1990).

13. G.V. Lavrelashvili, V.A. Rubakov, M.S. Serebryakov and P.G. Tinyakov, *Nucl. Phys.* **B329**, 98 (1990); V.A. Rubakov and P.G. Tinyakov, *Nucl. Phys.* **B342**, 430 (1990).

14. D. Scialom and P. Jetzer, *Phys. Rev.* **D51**, 5698 (1995).

15. B.L.Spokoiny, *Phys. Lett.* **129B**, 39 (1984); D.S. Salopek, J.R. Bond and J.M. Bardeen, *Phys. Rev.* **D40**, 1753 (1989); R. Fakir and W.G. Unruh, *Phys. Rev.* **D41**, 1783 (1990); R. Fakir, S. Habib and W.G. Unruh, *Aph. J.* **394**, 396 (1992); R. Fakir and S. Habib, *Mod. Phys. Lett.* **A8**, 2827 (1993).

16. A.O. Barvinsky, A.Yu. Kamenshchik and I.P. Karmazin, *Phys. Rev.* **D48**, 3677 (1993).

17. A.Yu. Kamenshchik, I.M. Khalatnikov and A.V. Toporensky, *Phys. Lett.* **B357**, 36 (1995).

18. K. Lee, *Phys. Rev. Lett.* **61**, 263 (1988).

19. S. Coleman and K. Lee, *Nucl. Phys.* **B329**, 387 (1990).

20. K. Lee, *Phys. Rev.* **D50**, 5333 (1994).

21. J.D. Brown, C.P.Burgess, A. Kshisagar, B.F. Whiting and J.W. York, *Nucl. Phys.* **B326**, 213 (1989); J. Twamley and D.N. Page, *Nucl. Phys.* **B378**, 247 (1992).

22. A.O. Barvinsky and A.Yu. Kamenshchik, *Phys. Rev.* **D50**, 5093 (1994); A.O. Barvinsky, A.Yu. Kamenshchik and I.V. Mishakov, *Nucl. Phys.* **491**, 387 (1997).

23. A.I. Baz', Ya.B. Zel'dovich and A.I. Perelomov, *Scattering, reactions and decays in non-relativistic quantum mechanics* (Nauka, Moscow, 1971).

24. R. Landauer and Th. Martin, *Rev. Mod. Phys.* **66**, 217 (1994).

25. A. Peres, *Am. J. Phys.* **48**, 552 (1980).

26. G.W. Gibbons and J.B. Hartle, *Phys. Rev.* **D42**, 2458 (1990).

Captions to Figures

FIG. 1. The geometries of the Euclidean regions at different values of the parameters: a) $Q = \Lambda = \lambda = \xi = 0, m \neq 0$ b)$Q = \lambda = \xi = 0, m \neq 0, \Lambda \neq 0$; c)$\lambda = \xi = 0, m \neq 0, \Lambda \neq 0, Q \neq 00$; d)$\lambda = 0, m \neq 0, \Lambda \neq 0, Q \neq 0, \xi \neq 0$; e)$\lambda = 0, \xi = \frac{16\pi^2 m^4 Q^2}{27 m_P^4}$; f)$\lambda = 0, \xi > \frac{16\pi^2 m^4 Q^2}{27 m_P^4}$; g)$\lambda = 0, \xi \gg \frac{16\pi^2 m^4 Q^2}{27 m_P^4}$; h)$\lambda \neq 0, \xi < \left(\frac{Q\lambda}{48}\right)^{2/3}$; i)$\xi > \left(\frac{Q\lambda}{48}\right)^{2/3}, \xi > \frac{16\pi^2 m^4 Q^2}{27 m_P^4}$.

FIG. 2. Boundaries between Euclidean and Lorentzian regions represented by short-dashed lines, instanton trajectories represented by bold lines, Lorentzian trajectories represented by long-dashed lines; $x-$ and $a-$ separating curves represented by thin lines.